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LENS DESIGNING TECHNIQUE WITH THE 1962 LASL CODE
FOR THE IBM 7090

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LENS DESIGNING TECHNIQUE WITH THE 1962 LASL CODE FOR THE IBM 7090

by

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ABSTRACT

This report is a detailed instruction manual for users of the 1962 LASL lens design code, which is a general-purpose computing program for the evaluation and automatic design of optical lens and/or mirror systems with surfaces generated by conic sections. Described in relation to the program are IBM-7090 machine operations, prescription parameters, ray-tracing parameters, weights on lens performance, lens-parameter substitutions, increments to lens parameters, designing instructions, and parameter interactions. The sample calculation for a Lister-type lens is given in great detail. The general procedure used to design a special-purpose zoom lens is given, along with the final lens prescription and the performance characteristics.

ACKNOWLEDGMENTS

My thanks go to John C. Holladay for developing the automatic lens-designing program and instructing me in its use, to Charles A. Lehman for developing this faster enlarged program, and to Mark B. Wells for making the calculations from which all but the smallest parameter calling sequences were discovered.

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INTRODUCTION

The purpose of this paper is to explain the use of the 1962 LASL automatic lens-designing code [1] and so to make optical lens designing available to any engineer or scientist with a modest knowledge of optics. The code gives to problems of lens design a general automated solution which makes full use of the digital computer's capabilities and abandons the classical approach requiring direction by the skilled specialist at every stage.

To study and design lenses, the code makes use of geometrical optics, the geometrical analysis of the light rays passing through the lens. Three laws of geometrical optics, those governing rectilinear propagation, refraction, and reflection, are used. By using these laws to trace light rays through a lens, we can analyze lens performance. The main criterion for lens performance is the image-spot size at the focal plane of interest. While image defects are analyzed precisely by evaluating the sizes and positions of image spots at the focal surface, the traditional Seidel aberrations are not evaluated. In specifying, selecting, or designing lenses for particular tasks,

this ability to analyze lens performance is invaluable.

The 1962 code developed by C. A. Lehman for the IBM-7090 Data Processing System is an adaptation of the automatic, iterative program developed in 1959 by J. C. Holladay for the IBM 704.[2] The LASL history of lens designing with computers began in 1950 when R. H. Stark applied floating-decimal computer notation to the LASL IBM-CPC computer.[3] Then W. A. Allen and Stark[4] made the first application to lens designing when they produced an easy to use ray-tracing program. In the same year R. E. von Holdt improved this program by substituting vectorial technique[5] for the algebraic methods used by Allen and Stark. These programs, however, limited themselves to ray tracing and required interpretation by an expert. Until 1959, attempts to broaden the system on the CPC and the Maniac met with indifferent success.

The Holladay program made statistical analyses of lens performance from skew ray traces. It determined differences in lens performance resulting from design-parameter alterations. Using these differences, it computed the linear combination which would give an improved lens prescription. With this system it was possible to design a complete lens by means of the computer. The success of this program created a desire for a code which would be both easier to use and able to solve more complex problems.

Consequently, Lehman has developed this faster code for the IBM-7090 Data Processing System. This new code can produce a lens design which is the result of a simultaneous analysis of as many as five different conditions of use. (A specific example is the zoom lens--to be described later--with five different focal lengths, apertures, and angles of view.) This code, in addition, can trace rays through cylindrical, displaced, and tilted surfaces, a feat not previously possible. Also new is simultaneous design for a maximum of six wave lengths. Use of the code is facilitated by the following automation improvements: design-parameter cycling, determinant adjustment, negative thickness prevention, and ray vignette informing.

The theory of this lens-designing program is treated in detail in Lehman's report[1] and will only be summarized in the following two sections. All information about lens performance is derived from rays traced by a skew ray-tracing routine. The deviation of the traced rays from their specified image points is an error that is reduced by alteration of the lens parameters as directed by the code's designing routine, which uses the least-squares technique. Since the subroutines for obtaining the square root and the linear matrix solution are well-known mathematical operations, they are not described. Also not described are the subroutines for on-line card reading,

on-line and off-line printing, punching binary prescription-data cards, and punching decimal point-plot cards.

RAY TRACING

Skew rays are traced through the lens or mirror system by the methods of geometrical optics. Computation is performed by the vectorial technique. The ray is traced in two stages to avoid the loss of accuracy which may occur in the machine solution of a quadratic equation when there is a long optical path. First a linear equation is used to locate the ray on the vertex tangent plane of the optical surface. Next the ray is traced to the optical surface and refracted or reflected. Then the unit vector for the refracted or reflected ray is calculated. This cycle of operations is repeated for each optical surface. Finally, the intersection of the ray with the focal surface (or other aiming point) is computed.

DESIGNING

Each ray that is traced through the lens system would intersect the specified image point if the lens gave perfect performance. In an imperfect lens the deviation of the ray from the specified image point is an error which can be weighed to make a weighed-error component. The sum of the weighed-error

components of all the traced rays is an error vector which is a measure of lens performance. To optimize the lens performance it is necessary to minimize the magnitude of the error vector by alteration of the lens parameters. The parameter alterations necessary to effect an improvement are determined by a least-squares method which is here applied by an arbitrary approximation of Newton's method of solving for minima of complicated functions. Selected design parameters of the lens are individually altered by a small amount, and the error vectors are computed for each. These error vectors are used to generate a matrix which is solved linearly to obtain an approximation of the parameter changes necessary to reduce the error vector and hence improve the performance of the lens.

Since the above solution is only an approximation of the desired error-vector minimum, only a modest parameter change is allowable, a maximum of perhaps 10 to 100 times the alterations used to make the computations. A maximum of 10 parameter increments can be calculated simultaneously, but experience indicates that on the preliminary design fewer should be used because of the increased dependence of the variables upon each other as the number increases. An iterative procedure is used to arrive at the minimum error vector obtainable with a specific set of parameter alterations.

WHAT THE PROGRAM CAN DO

The 1962 LASL lens designing code is a general-purpose computing program for the investigation of lens systems with as many as 98 surfaces. The program can determine the performance of an existing lens design. It can automatically improve an existing lens design to meet the conditions required by specific problems. And, finally, it can automatically obtain the optimum lens design from a start consisting of any desired arrangement of refracting and/or reflecting surfaces.

The lens or mirror surfaces most used are either spheres or cylinders but surfaces with any conic section may be used. The cylinder axis is always parallel to the x axis. Each of the surfaces of revolution may be displaced or tilted as desired. There is a wide choice possible in the number and distribution of the rays which may be traced through the lens. The positions and sizes of the entrance and exit pupils are readily controlled. The lens focus may be fitted to a specified focal surface, or the shape of the focal surface may be determined. The machine can design simultaneously for as many as seven object points, six wave lengths, and five different conditions of use.

The description which follows is a graphically illustrated exposition aimed at aiding the engineer to master the complexities of the use of the code. It has many numerical examples

of the data and of the program instructions which have been found useful for designing.

MACHINE OPERATION

This code is for an IBM-7090 Data Processing System[6] with the following components: a 7302 Core Storage (32,768-word), a 7606 Multiplexor, a 7607 Data Channel, SHARE II control panels, a 711-II Card Reader, a 716 Printer, a 721 Card Punch, a 729-IV Magnetic Tape Unit connected as logical tape A3, and a 1401 Data Processing System to interpret the tape record. All data are read from punched cards and must be addressed to cells specifically reserved for each item so that the operation programs can locate the specific data when needed.

The first operation which the operator must perform is to load the 213-card code into the card reader, where the program's coded instructions are read and sent to the core memory. This operation is followed by loading the lens data, which are on binary and/or decimal punched cards (columns 1 to 72). The lens data include the lens prescription and the parameters for ray tracing and designing. There are 10 individual program-control instructions on decimal punched cards, which are used to select the sequence of machine calculations.

All cards with decimal data and all operation instructions

are printed on line as soon as read. The tabulated lens data will be printed on line if the program discovers an error in the data. After completion of the calculations ordered by each program-control instruction, the specified answers and the latest lens prescription are recorded on the A3 tape. An on-line print of the tape record will also be obtained when sense switch six is down. The 721 Card Punch will make binary punched cards with the latest complete lens data (instruction *6 below [plus *3 and *4]) if sense switch one is down at the end of any of the ten routine program calculations. These lens data on binary punched cards are used to restart the problem for the next machine run. Lens-data changes can be read in before the start of any of the routine program calculations. The 1401 Processing Unit will, if ordered, supply the spot-diagram coordinates on decimal punched cards at the same time that the off-line print is made.

Decimal numerical (and alphabetic) data are stored in the machine's magnetic core memory after being read from punched cards and translated to binary notation. The cards are punched according to the conventions outlined below.

1. Integers prefixed by L or L- order the program to store the next number at that address (L-200 to L1999). Succeeding numbers go to succeeding addresses.

2. Integers prefixed by C order the program to clear that

many addresses in sequence.

3. Integers prefixed by X or X- are tally numbers.

4. Integers prefixed by * are program-control instructions.

5. Decimal numbers prefixed by + or - are positive or negative numbers representing the parameters. These numbers may contain as many as 10 digits and a decimal point. They may each be followed by a power-of-ten multiplier consisting of an integer (1 to 38) prefixed by E or E-.

6. The problem identification (any combination of machine characters and spaces heading each prescription print) placed between $'s will be stored as written (six per address) if prefixed by H and addressed to L-200 through L-171. The characters in L-200 and L-199 also appear on the print and the punched cards for spot-diagram plotting. ‡-200 TO -171

The program reads decimal punched cards automatically. Specified answers to calculations are printed automatically. The program control instructions and the calculations ordered by each are described below.

The symbol *1 instructs the program to read data on binary punched cards until a blank card is encountered, at which time the program returns to reading decimal punched cards.

*2 instructs the program to proceed with the lens designing (cycles and specifications scheduled at L-79 through L-65, L-49

‡ Marginal numbers are an index of storage addresses.

through L-3, and L1010 through L1499) and tabulate answers showing what has been accomplished.

*3 instructs the program to determine lens performance by multiple ray tracing (scheduled at L-170 through L-83, L-59 through L-56, and L1500 through L1999) and tabulate the answers giving the sizes and positions of the images.

*4 instructs the program to trace the upper and lower rim rays from the maximum object height and to tabulate the ray position and direction vectors and the distance from the preceding surface at each optical surface.

*5 instructs the program to trace paraxial and rim rays from zero object height and to tabulate the ray position and direction vectors and the distance from the preceding surface at each optical surface.

*6 instructs the program to make binary punched cards with the latest lens data.

*7 instructs the program to calculate and tabulate the scaled ray-intersection coordinates for spot-diagram plotting.

*8 instructs the program to calculate, tabulate, and make decimal punched cards with the scaled ray-intersection coordinates for spot-diagram plotting.

*9 instructs the program to tabulate the latest lens prescription.

*10 instructs the program to tabulate all of the lens data

that are in storage.

*0 instructs the program to <u>stop</u>, signifying the end of the run.

PRESCRIPTION PARAMETERS

To determine the performance of an existing lens pre-
scription we must know or find the wave-length (color) range,
the distance to the object, the angle of view, the entrance
pupil location, the relative aperture (f/number), the image
distance, the shape of the focal surface (plane or curved),
and (for some applications) the exit pupil location. The lens
illustrated in Figure 1 has each of the prescription parameters
labeled with its storage address number. Listed below are
descriptions of the storage nomenclature and the conventions
used as one starts at the first surface and proceeds through
the lens. The lens prescription is stored in the memory by
addressing the parameters as described below. The radii of
curvature are sent to L32, L42, etc. Surfaces which appear 32
convex as seen from the left are positive; concave ones are
negative; and the symbol for a plane is +0. The axial distances
or thicknesses from the previous surfaces are sent to L31, L41, 31
etc. Distances proceeding to the right are positive and to the
left, negative. The first distance (L31) is from the entrance

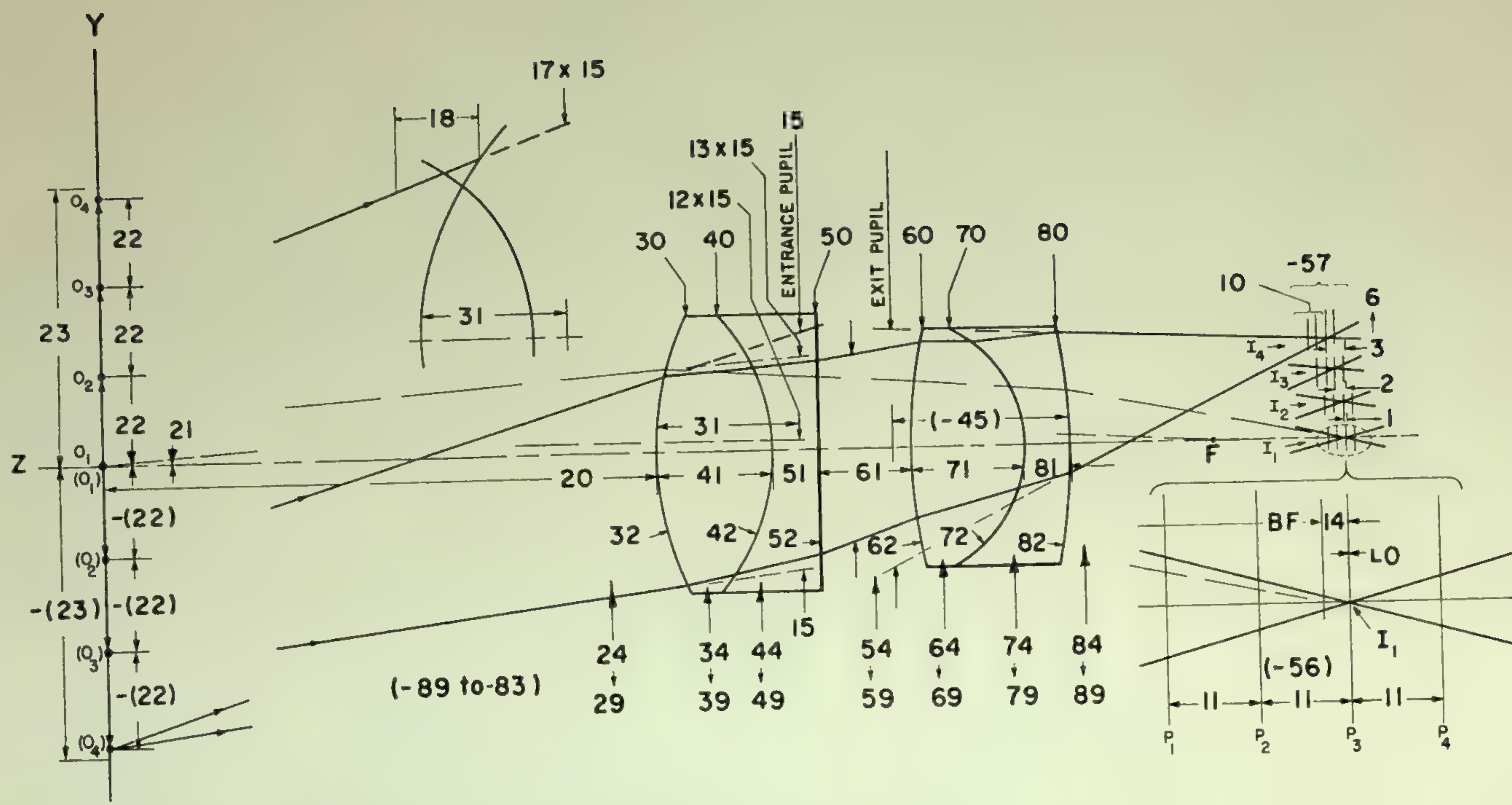

Figure 1. Lens optical diagram showing prescription parameter storage addresses.

pupil. With the pupil to the left the distance is positive, to the right it is negative. The aperture radii of the surfaces are sent to L30, L40, etc. If the radii are not specified the symbol +0 is used, and the code will not check for ray vignettes from this cause. Central obstructions are specified by negative radii. 30

The optical surfaces are generated by conic sections. The eccentricities which describe the conic sections are sent to L33, L43, etc. (0 for a sphere, 1 for a parabola, >1 for a hyperbola, between 1 and 0 or negative for axially thick and axially thin ellipses, respectively). 33

The refractive indexes for the first wave length (color) are sent to L24 (object space index), L34 (first glass index), L44, etc. For the second wave length the indexes go to L25, L35, etc. Indexes for a total of six wave lengths may be used (to L29, L39, etc.). The refractive indexes may be used in any order and number desired by storing in L-89 the number of colors to be used, the number of the first color to be used in L-88, the second color in L-87, etc. (example: L-89 X3 X2 X1 X3). 24 25 TO 29 -89 -88 TO -83

For a displaced and/or tilted spherical or conoid surface the aperture radius, the axial distance from the previous surface, the radius of curvature, and the eccentricity are stored at the usual addresses. However, some variation of

the normal procedure is necessary. Twenty storage addresses are required. In place of the first index of refraction, +0 is stored, a signal that more surface data are to follow. Plus one is stored in the next address, a signal that displacement and/or tilt parameters will be given. The next four addresses are cleared. The next 10 addresses have the x translation, the y translation, the sine of the angle of tilt in the x direction, the sine of the angle of tilt in the y direction, and the six indexes of refraction. (Example: L30 +0 +1.2 +4.65 +1 +0 +1 C4 +.015 -.01 +.03490 +.01745 +1.51462 +1.51700 +1.51901 +1.52264 +1.52712 +1.53047)

A cylindrical surface (whose axis is parallel to the x axis and normal to the optical axis) requires a procedure which is similar to that specified for displaced and tilted spherical surfaces. The aperture radius, the axial distance from the previous surface, the radius of curvature, and the eccentricity are stored at the usual addresses. Twenty storage addresses are required. In place of the first index of refraction, +0 is stored, a signal that more surface data are to follow. Plus two is stored at the next address, a signal that this surface is cylindrical. The next eight addresses are cleared, and the six indexes of refraction follow. (Example: L30 +0 +1.2 +4.65 +1 +0 +2 C8 +1.51462 +1.51700 +1.51901 +1.52264 +1.52712 +1.53047)

When a mirror surface is to be specified, a sign change is made in the refractive index of the medium which follows the mirror surface. The changed sign will be kept by succeeding refractive media until another mirror surface occurs, at which time the sign of the refractive index is again changed. To specify surface radii and distances in mirror systems the same conventions are used as for refracting systems.

RAY-TRACING PARAMETERS

The distance to the object and the maximum angle of view are determined by the design specifications, or, in the absence of specific claims, the object distance is assumed to be infinity (up to $\pm 10^{38}$), and the field angle is discovered by tracing series of ray bundles from object points at increasing angles. The object distance from the first lens surface is
stored at L20. The angle of view is determined by the object 20
height. The initial object height is stored at L21, the 21
object-height increment is stored at L22, and the maximum 22
allowable object height at L23. 23

If the image distance is specified, it is stored at L14 14
and the plane setting mode X1 stored at L-56. If the image -56
field is curved, the additional sagitta for the first, second,
etc., image points are stored at L0 to L6. If the image dis- 0 TO 6

tance is unknown, zero is stored at L14 and the plane setting mode X0 is used. The ray-tracing program will then set the planes of interest at the back focus calculated from the meridional ray trace plus L14 plus the corresponding sagitta in L0 to L6.

The distance of the entrance pupil from the first surface
31 is stored at L31, negative if to the right and positive if to
the left. If the entrance pupil distance is not specified it
may be set at an estimated distance.

The desired f/number of the image beams is stored in the
-49 weight section at L-49. The symbol +0 is stored at L-49 if
one wishes to fix the entrance-pupil radius when designing.
If the f/number is known, the entrance pupil radius is hand
15 computed and stored at L15.

The numbers which designate the shape of the focal surface are stored in L0 to L6 and are zero if the focal surface is a plane. For a curved focal surface the numbers are positive or negative for convex or concave shapes, respectively.

The image spot is evaluated from the size of the ray-
bundle intersection on a plane normal to the optical axis. The
predicted resolution is the mean deviation of the rays from
their mean center. Any number of planes may be selected by
-57 storing the number desired in L-57 (example: X3). The spacing
between planes will be determined by the number stored in

L11 (example: +.01). The location of the first plane to be 11
printed can be shifted by storing a number in L10 (example: -1 10
sets the first plane one to the left of the plane of interest).

The (paraxial) focal length is computed from a parallel
ray traced automatically at an aperture height determined by
the number stored in L12 (example: +.00001) multiplied by 12
the maximum aperture stored in L15.

The back focal length is computed from an axial-object
ray traced automatically at an aperture height determined by
the number stored in L13 (example: +.707) multiplied by the 13
maximum aperture stored in L15.

Feathering or negative thickness between adjacent lens
surfaces is calculated with a paraxial ray and a marginal ray
for the axial object position and with the upper and lower
marginal rays for the extreme off-axis object position when
the entrance pupil scale factor +1 is stored in L17 and a 17
minimum feather thickness of +0 is stored in L18. To avoid 18
machine stops, which may occur if feathering develops, the
scale factor and thickness values may be changed to smaller
and negative amounts, respectively, at the convenience of
the designer.

The pattern of rays to be traced is determined by the
numbers stored in L-170 through L-90. The pattern of rays is -170 TO -90
placed in a unit-radius entrance pupil. The ray-trace program

scales the unit-pupil dimensions to the actual pupil dimensions stored in L15 to obtain the coordinates of each ray to be traced. The rays are usually specified to sample the entrance pupil with a grid pattern in which each ray represents the surrounding area, but as many as 25 rays may be positioned individually in any pattern desired. Since rays traced through a centered optical system are symmetrical, only half the rays of symmetrical ray patterns need to be traced to obtain the ray intersections of the other half by a sign change of the
-170 x coordinate. Mode X1 stored in L-170 directs the program to trace the specified rays only and to generate the symmetrical ray intersections by a sign change in the xs ($\bar{x} \equiv 0$). Mode X0 stored in L-170 directs the program to trace the specified rays with coordinates (x,y) and also rays with coordinates (-x,y) when uncentered optical systems are being considered ($\bar{x}$ is calculated). Mode X2 stored in L-170 directs the program to trace the specified rays only through centered or uncentered systems ($\bar{x}$ is calculated). The total number of rays to be
-169 traced in the ray pattern (100 maximum) is stored in L-169.
-168 The vertical ray spacing (Δy) is stored in L-168.

The ray coordinates are now generated in columns by triads of numbers consisting of the x coordinate of the column of
-167 rays (stored in L-167, L-164, . . . L-92), the y coordinate
-166 of the first ray of the column of rays (stored in L-166, L-163,

. . . L-91), and the number of rays in the column (stored in L-165, L-162, . . . L-90). -165

Figure 2 illustrates, and Table 1 gives the instructions for generating, the ray patterns which have been used for lens designing and for testing the performance of lenses. The two-ray pattern is especially useful during the preliminary design stage because it is economical of machine time while still evaluating adequately the major lens defects for most design problems. The six-ray pattern is usually adequate for the remainder of the designing, but a final run with the 12-ray pattern may occasionally greatly improve lens performance. For assessing lens performance, the many-ray patterns are especially useful when there is a widely variable vignetting of the beams which form the several images.

The ray pattern on any specified plane may be graphed from the ray-intersection coordinates obtained from prints or punched cards which are supplied at the scale required by the point-plot factor stored in L16 (example: +100). 16

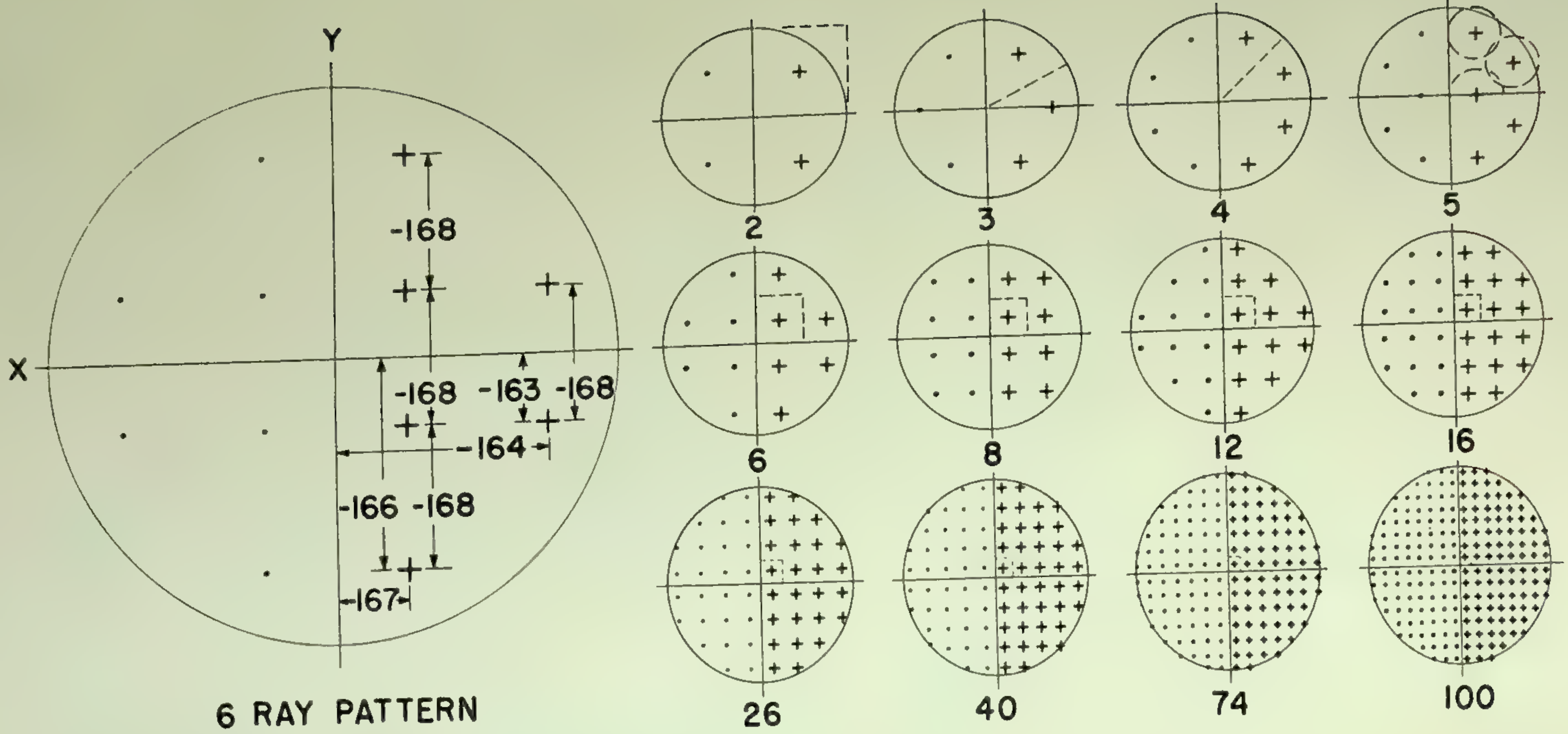

Figure 2. Ray-pattern examples and sample ray-coordinate addresses.

TABLE 1

EXAMPLES OF INSTRUCTIONS FOR GENERATING SELECTED RAY PATTERNS

Rays	Card Image		
2	L-170 X1 X2 +1	+.5 -.5 X2	
3	L-170 X1 X3 +1.2	+.35 -.6 X2	+.7 +0 X1
4	L-170 X1 X4 +1.4	+.3 -.7 X2	+.7 -.3 X1
	+.7 +.3 X1		
5	L-170 X1 X5 +.7	+.3 -.7 X3	+.7 -.35 X2
6	L-170 X1 X6 +.5	+.25 -.75 X4	+.75 -.25 X2
8	L-170 X1 X8 +.4	+.2 -.6 X4	+.6 -.6 X4
12	L-170 X1 X12 +.35	+.175 -.875 X6	+.525 -.525 X4
	+.875 -.175 X2		
16	L-170 X1 X16 +.3	+.15 -.75 X6	+.45 -.75 X6
	+.75 -.45 X4		
26	L-170 X1 X26 +.25	+.125 -.875 X8	+.375 -.875 X8
	+.625 -.625 X6	+.875 -.375 X4	
40	L-170 X1 X40 +.2	+.1 -.9 X10	+.3 -.9 X10
	+.5 -.7 X8	+.7 -.7 X8	+.9 -.3 X4
74	L-170 X1 X74 +.15	+.075 -.975 X14	+.225 -.975 X14
	L-161 +.375 -.825 X12	+.525 -.825 X12	+.675 -.675 X10
	+.825 -.525 X8	+.975 -.225 X4	
100	L-170 X1 X100 +.125	+.0625-.9375X16	+.1875-.9375X16
	L-161 +.3125-.9375X16	+.4375-.8125X14	+.5625-.6875X12
	L-152 +.6875-.6875X12	+.8125-.4375X8	+.9375-.3125X6

LENS-DESIGNING PARAMETERS

The performance of a particular prescription is influenced by the position of the entrance pupil, the position of the exit pupil, the position of the focal surface, and the rays vignetted at the field edge. The optimum positions of the first three can be determined by the designing part of the code. Since ray vignettes from any cause will stop the designing sequence, the effect of vignettes can only be tested with the ray-tracing part of the code.

To use the designing program it is necessary to provide (1) weights for the performance errors, (2) substitutions (if desired) for the lens parameters, (3) increments to the desired lens parameters, and (4) designing instructions.

WEIGHTS

The operating relative aperture (*f*/number) of the lens
-49 is stored in L-49. The entrance pupil radius will then be
recalculated before each design cycle. If the entrance pupil
radius is to be fixed, zero is stored in L-49. The required
-48 principal focal length is stored in L-48. The weight on the
-47 focal length (if any) is stored in L-47. The exit-pupil posi-
tion is calculated from a ray traced from an object point at

a height stored in L-45. The desired distance of the exit -45
pupil from the last optical surface is stored in L-44. The -44
weight on the desired exit-pupil distance is stored in L-43. -43

Optimum performance generally requires minimum image-spot sizes on the plane of interest. These image spots are made up of a maximum of six colors each from a maximum of seven
object points. The weights for these spots are stored in L-39 -39 TO
to L-33. Weights on the lateral chromatic aberration for each -33
of these seven images are stored in L-29 to L-23. Weights on -29 TO
the desired image height for the average height of each of the -23
seven images are stored in L-19 to L-13. The seven desired -19 TO
image heights are stored in L-9 to L-3. The above weights -13 -9
are the total number which control the design program. The TO -3
number stored in L-46 determines another control factor, the
maximum parameter changes allowed by the program. To obtain -46
the individual limits, this number is multiplied by each parameter alteration.

SUBSTITUTIONS

The code permits the automatic designing of a lens in as many as five conditions of use (for example a zoom lens) by means of a substitution routine. The addresses (L-200 to
L1499) of the parameters to be substituted are stored in L1500 1500

to L1599 (example: L1500 X14 X-49 X51). The corresponding
1600 parameters to be substituted first are stored in L1600 to
L1699 (example: L1600 +350 +6 +100). The second set of
1700 parameters is stored in L1700 to L1799 (example: L1700 +400
1800 +7 +150). The third set is stored in L1800 to L1899 and the
1900 fourth set in L1900 to L1999. The substitutions will be made
and designing accomplished automatically if the number stored
-59 in L-59 is the number of sets of substitution parameters and
-58 the number stored in L-58 is the number of substitution param-
eters in each set. (example: L-59 X2 X3).

INCREMENTS

Any of the prescription parameters (L0 to L1999) may be altered by the design program to give optimum lens performance. Storage has been provided for the design of as many as 49 individual parameters. As many as six parameters can be given identical alterations simultaneously in any plus or minus combination. The increment instruction requires 10 storage
1010 cells, starting at L1010, L1020, . . . L1490. The first num-
ber, X1 to X6 or X-1 to X-6, is the number of addresses on which this increment is used. The sign of this number is a convention. If it is positive, the increment is added to the parameter; if negative, the increment is added to the recip-

rocal of the parameter (for radius of curvature). The second
number, X0 to X1999 or X-0 to X-1999, is the positive address 1011
only of the first parameter which uses this increment. The
sign of this and the next five numbers is a convention. If
it is positive, the increment is added to the parameter or its
reciprocal; if negative, the increment is subtracted from the
parameter or its reciprocal. The third number, X0 to X1999 1012
or X-0 to X-1999, is the address of the second parameter which
uses this increment. The fourth, fifth, sixth, and seventh 1013 TO
numbers are the addresses of the third, fourth, fifth, and 1016
sixth parameters which use this increment. All parameters
addressed here will be given the same amount of alteration, a
capability useful for the design of symmetrical systems and
also for holding selected lengths or curvatures equal or con-
stant as desired. The number stored in the next cell is the
program-computed single determinant for the increment used. 1017
The next number is the single determinant specified for this 1018
increment by the designer. The last number is the actual 1019
increment used by the design program. The minimum value for
any increment (example: .000001) is stored at L19 and is 19
automatically transferred to L1019, L1029, etc., if the incre-
ment calculated from the required determinant is less than
the assigned minimum. Five examples of parameter-increment
instruction card images are shown in Table 2.

TABLE 2

EXAMPLES OF PARAMETER-INCREMENT INSTRUCTIONS

L1010	X-1	X32	L1018	+.1	+.0001					
L1010	X-1	X32	C6	+.1	+.0001					
L1020	X2	X51	X71	C5	+.01	+.1				
L1030	X-4	X42	X52	X-62	X-72	C3			+.1	+.0001
L1040	X-6	X32	X42	X52	X-62	X-72	X-82	C1	+.1	+.00001

The increment-instruction sequences assume the serial identifications 1, 2, 3, etc., which are used to call any one or any continuous series of them for designing.

DESIGNING INSTRUCTIONS

The design-parameter increment-instruction sequences described above can be used only if the required instructions
-79 are stored in L-79 to L-65. The minimum identification number
-78 to be used is stored in L-79 and the maximum in L-78. The
-77 number stored in L-77 is the number of times this set of increment cycles is to be used for designing. The total number (1 to 10 permissible) of parameter increment identifications
-76 to be used in each cycle is stored in L-76. The sequence of
-75 TO identification numbers to be used is next stored in L-75 to
-66 L-66. With each design cycle each number in the calling sequence is incremented by 1 (any becoming larger than the

maximum will be reset to the minimum) so that all parameters in a selected series will be used in serial order and repeated if enough design cycles are ordered. This arrangement is used because it is frequently not possible or desirable to use all parameters at one time, yet all must eventually be allowed to work the prescription. It has been found advantageous to arrange the increment calling sequence so that one obtains the greatest variety of interactions between parameters. With crude starting prescriptions it is often found that the single and multiple determinants tend to fluctuate violently, and for this reason designing is usually started with only one or two increments per cycle. As the design is stabilized the number of increments per cycle is increased. Most designing has been done with the single-increment determinant in the 10^{2} to 10^{-2} range. With eight increments being used simultaneously, the determinant frequently ranges from 10^{-5} to 10^{-25}. When the determinant is in the 10^{-25} to 10^{-35} range, there is often great fluctuation in the design progress. If a particular combination of increments does not improve the prescription, the use of that combination will be skipped for the same number of cycles as the number stored in L-65. Six examples of -65
design-cycle instruction card images are shown in Table 3.

TABLE 3

EXAMPLES OF DESIGN-CYCLE INSTRUCTIONS

L-79	X1	X12	X25	X1	X1	C9								
L-79	X1	X12	X37	X2	X1	X5	C8							
L-79	X1	X12	X18	X4	X1	X2	X4	X8	C6					
L-79	X1	X12	X18	X6	X1	X2	X3	X5	X7	X10	C4			
L-79	X1	X14	X29	X6	X1	X2	X3	X4	X8	X11	C4			
L-79	X1	X14	X29	X8	X5	X6	X7	X9	X12	X13	X14	X2	C2	X1

PARAMETER INTERACTIONS

Because of the interactions among parameters, the design-progress rate increases with the number of increments working simultaneously. The number of interactions between n parameters working simultaneously is $\frac{1}{2}n(n-1)$. (For example, with this program one obtains 0, 1, 3, 6, 10, 15, 21, 28, 36, or 45 parameter interactions depending on the number of increments worked simultaneously.) The design-progress rate is increased as the number and variety of the parameter interactions increase. The working (incremented) parameters are serially stored, and the calling order is at the command of the designer. For example, with a series of 15 parameters in which five are being worked simultaneously, one might select every third parameter, and the three calling sequences for designing would be 1, 4, 7, 10, 13; 2, 5, 8, 11, 14; and 3, 6, 9, 12, 15. All

parameters are used every three design cycles, but only two varieties of parameter interaction are present, i.e., those spaced 3 and 6 apart. At the other extreme one can select the 15 calling sequences: 1, 2, 3, 4, 5; . . . ; 15, 1, 2, 3, 4. In this case all parameters are worked every 11 design cycles, with four varieties of parameter interaction present. Or one might use the sequences 1, 2, 3, 5, 14; 2, 3, 4, 6, 15; . . . . Here all parameters are worked every nine design cycles, with five varieties of parameter interaction present. Or the reverse sequence 1, 3, 12, 14, 15; . . . would be equally useful. However, the greatest variation of interaction is obtained with the 15 calling sequences 1, 2, 5, 10, 12; 2, 3, 6, 11, 13; . . . , with all parameters worked every five design cycles, and all seven possible varieties of parameter interaction present. For clarity this information is summarized in Table 4.

TABLE 4

ANALYSIS OF PARAMETER INTERACTION IN SELECTED SEQUENCES

Design Parameter Calling Sequence	Spaces Between Parameters					Parameter Spacing Incidence Frequency						
	1st	2nd	3rd	4th	5th	1	2	3	4	5	6	7
1, 4, 7, 10, 13	3	3	3	3	3	0	0	5	0	0	5	0
1, 2, 3, 4, 5	1	1	1	1	11	4	3	2	1	0	0	0
1, 2, 3, 5, 14	1	1	2	9	2	2	3	2	2	0	1	0
1, 3, 12, 14, 15	2	9	2	1	1	2	3	2	2	0	1	0
1, 2, 5, 10, 12	1	3	5	2	4	1	1	1	2	2	1	2

Table 5 gives an extensive series of parameter calling sequences, which have been selected in each case to provide all possible parameter interactions. The number of interactions of each type is distributed as evenly as possible, and each parameter is repeated as often as possible by avoiding sequences with large parameter spacings if there is a choice.

SAMPLE CALCULATION

The lens to be designed in the following sample calculation is similar to the lens illustrated in Figure 1. It consists of two cemented doublets in series which produce an $f/2$ Lister-type lens[7] set to work at a magnification of about one third. (To increase the field of view and field curvature for this illustration, the space between the two components was reduced.) The objects are at an axial distance of 12 inches from the first lens surface and 0, 1, 2, and 3 inches from the optical axis. The images are on a curved field which is held at an axial back focal distance of 3 inches by the use of plane setting mode X1. The design was started with two cemented sheets of 651584 and 720362 glass spaced 1 inch apart. The entrance pupil distance was fixed at -1 inch and the aperture radius at 1 inch. Figure 3 shows the complete series of card images used to design this lens. The data for the starting prescription, the weights, the ray patterns, the parameter

TABLE 5

EXAMPLES OF EFFICIENT PARAMETER CALLING SEQUENCES

Total No. Parameters	Number of Parameters Used Simultaneously 4	5	6	7	8	9	10
6	1,2,4,5						
7	1,2,3,5	1,2,3,5,6					
8	1,2,4,6	1,2,3,5,6	1,2,3,5,6,7				
9	1,2,5,7	1,2,4,6,7	1,2,3,5,6,7	1,2,3,4,6,7,8			
10	1,2,4,7	1,2,3,6,8	1,2,3,5,6,8	1,2,3,4,6,7,9			
11	1,2,4,8	1,2,3,5,8	1,2,3,5,6,8	1,2,4,5,6,7,9	1,2,3,4,6,7,8,9		
12	1,2,4,8	1,2,3,6,9	1,2,3,5,7,10	1,2,4,6,7,8,10	1,2,3,4,5,7,8,10	1,2,3,4,5,6,7,9,10	1,2,3,4,5,7,8,9,10,11
13	1,3,4,8	1,2,5,8,10	1,2,5,7,8,10	1,2,3,5,6,9,11	1,2,3,4,6,7,8,10	1,2,3,4,5,7,8,9,11	1,2,3,4,5,7,8,9,11,12
14	1,3,7,10	1,2,4,7,11	1,2,3,4,8,11	1,2,4,7,8,9,11	1,2,3,5,7,8,10,11	1,2,3,5,6,7,8,9,11	1,2,3,4,6,7,9,10,11,13
15	1,3,6,10	1,2,4,8,12	1,2,3,5,8,12	1,2,3,5,6,9,11	1,2,3,5,8,9,10,12	1,2,4,5,6,8,9,11,13	1,2,5,6,7,8,9,10,11,13
16	1,3,8,11	1,2,4,8,12	1,3,4,5,9,12	1,2,3,5,7,10,13	1,2,3,4,6,8,9,12	1,2,5,6,7,8,9,11,13	1,2,3,4,5,6,9,10,12,14
17	1,3,8,12	1,2,4,8,13	1,2,4,7,10,14	1,2,3,4,7,10,14	1,2,3,6,8,10,11,14	1,2,3,4,6,9,10,12,14	1,2,3,4,5,6,9,10,13,15
18	1,5,8,13	1,3,4,8,13	1,2,3,5,9,14	1,2,6,8,9,11,15	1,2,3,4,5,7,10,14	1,2,5,6,7,8,10,12,15	1,3,4,5,6,9,10,12,14,16
19	1,4,9,13	1,3,4,8,14	1,2,4,7,11,15	1,3,4,5,7,10,15	1,2,3,4,6,10,12,15	1,2,5,7,8,9,11,14,16	1,2,4,5,8,9,10,11,13,15
20	1,4,8,13	1,4,9,11,15	1,3,4,7,11,16	1,2,7,9,11,12,15	1,2,3,4,7,9,12,16	1,2,5,8,9,10,11,13,15	1,3,4,7,9,11,12,13,14,17
21	1,6,9,15	1,3,8,9,12	1,2,4,8,12,17	1,2,3,5,9,12,17	1,3,6,7,8,11,14,17	1,3,4,7,9,12,13,14,17	1,4,5,6,7,10,11,13,15,18
22	1,5,10,16	1,2,5,11,16	1,2,4,8,13,18	1,2,3,5,9,13,18	1,4,5,6,10,12,15,18	1,3,7,9,10,12,13,14,17	1,2,3,5,8,12,13,14,16,19
23	1,5,10,16	1,2,4,9,15	1,2,4,9,13,19	1,2,3,5,9,14,19	1,2,3,4,7,10,14,18	1,2,3,5,9,13,14,16,19	1,2,3,4,5,8,12,14,17,20
24	1,5,10,17	1,2,5,12,17	1,2,4,9,13,19	1,2,3,5,9,14,19	1,2,3,4,6,10,14,19	1,2,3,4,6,9,12,16,20	1,2,3,5,7,12,14,15,17,21
25	1,5,10,16	1,2,5,12,18	1,2,3,6,12,19	1,2,3,5,10,14,20	1,2,3,4,8,13,16,20	1,2,3,4,7,12,15,19,21	1,2,3,4,6,10,14,17,20,22
26	1,6,13,21	1,2,6,12,19	1,2,4,11,15,22	1,2,3,5,10,15,21	1,2,3,5,8,13,17,21	1,2,3,5,8,13,17,20,22	1,2,3,4,6,9,13,15,19,22
27	1,6,13,22	1,2,5,13,19	1,2,4,6,14,22	1,2,4,8,12,17,22	1,2,3,4,9,13,17,22	1,2,3,5,9,14,16,19,22	1,2,3,4,8,10,13,16,20,24
28	1,6,12,20	1,2,5,13,20	1,2,5,16,21,23	1,2,4,9,13,17,23	1,2,3,4,7,11,17,22	1,2,3,4,6,10,13,18,24	1,2,3,4,6,9,15,16,20,24
29	1,6,13,21	1,2,6,13,21	1,2,3,7,14,22	1,2,3,5,11,16,23	1,2,3,5,8,12,17,23	1,2,3,5,8,11,15,19,24	1,2,3,4,6,10,15,19,22,25
30	1,7,14,22	1,2,7,14,22	1,2,3,6,15,25	1,2,3,6,11,17,24	1,2,3,5,8,13,17,23	1,2,3,4,6,9,14,18,24	1,2,3,4,7,11,17,19,22,26
					1,2,4,12,13,17,25,27	1,2,3,4,6,10,16,20,25	

```
L-198 H$SAMPLE PROBLEM FOR ILLUSTRATION      $
L-192 H$ 01--30--63 $
L-89 X5 X1 X2 X3  L-57 X4 X1  L-46 +20  L-39 +100 +100 +100 +100
L10 -2 +.1 +.0001 +.707 +3 +1 +1000 +1 -1 +.0001
L20 +12 +0 +1 +3.1 +1 +1 +1  L30 +0 -1 +0 +0 +1.651 +1.64771 +1.65885
L40 +0 +1.25 +0 +0 +1.72 +1.71434 +1.73422  L50 +0 +.5 +0 +0 +1 +1 +1
L60 +0 +1 +0 +0 +1.651 +1.64771 +1.65885
L70 +0 +1.25 +0 +0 +1.72 +1.71434 +1.73422  L80 +0 +.5 +0 +0 +1 +1 +1
L1010 X-1 X32 C6 +.1 +.01  L1020 X-1 X52 C6 +.1 +.01 X-1 X62 C6 +.1 +.01
L1040 X-1 X82 C6 +.1 +.01  X-1 X42 C6 +.1 +.01  X-1 X72 C6 +.1 +.01
L1070 X1 X1 C6 +.05 +.1  X1 X2 C6 +.05 +.1  X1 X3 C6 +.05 +.1
L1100 X1 X31 C6 +.05 +.1
L-170 X1 X2 +1 +.5 -.5 X2              L-79  X1 X4  X9  X1  X1  C9   +2
L-79  X1 X6  X13  X1  X1  C9   +2
L-79  X1 X9  X19  X1  X1  C9   +2
L-49 +2   L-79  X1 X9  X19  X2  X1 X4  C8   +2
L-79  X1 X9  X28  X4  X1 X2 X5 X7  C6   +2
L-170 X1 X6 +.5 +.25 -.75 X4 +.75 -.25 X2   L-79X1X9X10X4X1X2X3X5C6  +2
L-79  X1 X10  X11  X6  X1 X2 X3 X4 X6 X8  C4  +2
L-170 X1 X12 +.3333 +.1667 -.8333 X6 +.5 -.5 X4 +.8333 -.1667 X2
L-79  X1 X10  X5  X6  X1 X2 X3 X4 X6 X8 C4   +2
+3
+4
+6
+5
+0


PRESCRIPTION                              SAMPLE PROBLEM FOR ILLUSTRATION                            01--30--63

10  -2.0000000+00   1.0000000-01   9.9999975-05   7.0699999-01   3.0000000+00   1.0000000+00   1.0000000+03
                                                                 1.0000000+00  -1.0000000+00   9.9999975-05
20   1.2000000+01   0.0000000+00   1.0000000+00   3.1000000+00   1.0000000+00   1.0000000+00   1.0000000+00
30   0.0000000+00  -1.0000000+00   0.0000000+00   0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
40   0.0000000+00   1.2500000+00   0.0000000+00   0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
50   0.0000000+00   5.0000000-01   0.0000000+00   0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
60   0.0000000+00   1.0000000+00   0.0000000+00   0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
70   0.0000000+00   1.2500000+00   0.0000000+00   0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
80   0.0000000+00   5.0000000-01   0.0000000+00   0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
  PLANE SETTING MODE O.E.      LATTICE MODE ONE.        SENSE SWITCH MODE ZERO.        NUMBER OF PLANES   8

  WEIGHTS
 -49    0.0000000+00   0.0000000+00   0.0000000+00   2.0000000+01   0.0000000+00   0.0000000+00   0.0000000+00
 -59    1.0000000+02   1.0000000+02   1.0000000+02   1.0000000+02   0.0000000+00   0.0000000+00   0.0000000+00
 -29    0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00
 -19    0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00
  -9    0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00

   LATTICE N =  2      DELTA Y =  1.0000000+00
              5.0000000-01  -5.0000000-01     2

  INCREMENT IDENTIFICATION...  1     4     9     1     1

  COLOR IDENTIFICATION...      3   1   2   3

   ID.    LOC.     NO.        ADDRESSES OF DESIGN PARAMETERS     ACTUAL DET.    REQUIRED DET.    INCREMENT
    1     1010     -1     32                                     0.0000000+00   1.0000000-01   9.9999999-03
    2     1020     -1     52                                     0.0000000+00   1.0000000-01   9.9999999-03
    3     1030     -1     62                                     0.0000000+00   1.0000000-01   9.9999999-03
    4     1040     -1     82                                     0.0000000+00   1.0000000-01   9.9999999-03
    5     1050     -1     42                                     0.0000000+00   1.0000000-01   9.9999999-03
    6     1060     -1     72                                     0.0000000+00   1.0000000-01   9.9999999-03
    7     1070      1      1                                     0.0000000+00   5.0000000-02   1.0000000-01
    8     1080      1      2                                     0.0000000+00   5.0000000-02   1.0000000-01
    9     1090      1      3                                     0.0000000+00   5.0000000-02   1.0000000-01
   10     1100      1     31                                     0.0000000+00   5.0000000-02   1.0000000-01

             1  2  3  4  5  6  7  8  9 10   INCR. FACTOR   DETERMINANT    OLD LENGTH     NEW LENGTH     IMPROVEMENT

  MERIT     1                                5.4053847-01   8.4568026+01   3.4037419+02   1.5558818+02   5.4289076-01

  MERIT     2                                1.0000000+00   6.5409193+01   1.5558818+02   1.1117392+01   9.2854603-01

  MERIT     3                                1.0000000+00   2.1871955+01   1.1117392+01   9.9965106+00   1.0082233-01

  MERIT     4                                1.0000000+00   7.8169171+00   9.9965106+00   9.9836626+00   1.2852504-03

  MERIT     1                                1.0000000+00   8.7207531-02   9.9836626+00   9.9557289+00   2.7979428-03

  MERIT     2                                1.0000000+00   1.1868808-01   9.9557289+00   9.9545368+00   1.1973739-04

  MERIT     3                                1.0000000+00   1.0032179-01   9.9545368+00   9.9462004+00   8.3744979-04

  MERIT     4                                1.0000000+00   1.0070031-01   9.9462004+00   9.9337771+00   1.2490475-03

  MERIT     1                                1.0000000+00   9.9397428-02   9.9337771+00   9.9058612+00   2.8101934-03

PRESCRIPTION                              SAMPLE PROBLEM FOR ILLUSTRATION                            01--30--63

10  -2.0000000+00   1.0000000-01   9.9999975-05   7.0699999-01   3.0000000+00   1.0000000+00   1.0000000+03
                                                                 1.0000000+00  -1.0000000+00   9.9999975-05
20   1.2000000+01   0.0000000+00   1.0000000+00   3.1000000+00   1.0000000+00   1.0000000+00   1.0000000+00
30   0.0000000+00  -1.0000000+00   5.0438474+00   0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
40   0.0000000+00   1.2500000+00   0.0000000+00   0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
50   0.0000000+00   5.0000000-01  -5.2206752+00   0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
60   0.0000000+00   1.0000000+00  -1.0487488+02   0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
70   0.0000000+00   1.2500000+00   0.0000000+00   0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
80   0.0000000+00   5.0000000-01  -2.8214286+02   0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
```

Figure 3. Printed output showing Lister lens design start.

alterations, the design instructions, and the machine operations are given. This information is followed by the complete prescription data print, the accomplishments of each of the next nine design cycles, and the resulting lens prescription.

Designing was started with three colors (C, D, F), a two-ray pattern, and a series of nine single increments to the four glass-air surfaces. As a result the prescription's figure of merit went from 340 to 9.9. A summary of the complete 9-minute series of runs used to design this lens is shown in Table 6.

TABLE 6

SUMMARY OF LISTER LENS DESIGNING SEQUENCE AND PROGRESS

Operation Performed	Ray Pattern	Increments Used		Cycles Run	Figure of Merit
		Total	Per Cycle		
Start	2	-	-	-	340.
Designing	2	4	1	9	9.9
Designing	2	6	1	13	9.7
Designing	2	9	1	19	5.9
$f/2.0$	2	-	-	-	15.4
Designing	2	9	2	19	6.6
Designing	2	9	4	28	1.6
Ray Change	6	-	-	-	1.7
Designing	6	9	4	10	1.2
Designing	6	10	6	11	0.96
Ray Change	12	-	-	-	0.96
Designing	12	10	6	5	0.94

These runs produced an $f/2.0$ lens of 3.461-inch (D-light) focal length, which should give 400-lines-per-inch resolution on a 3.0525-inch-radius concave focal surface. There is a maximum of 0.002 inch lateral chromatic aberration at the edge of the 13-degree half field of view. Figure 4 shows the print of the final prescription and the 12-ray analysis of lens performance. The spot diagrams of the four images are shown in Figure 5.

A COMPLEX DESIGN

An example of a complex design which can be made with this program is a special-purpose zoom lens. This lens is an objective for a high-speed camera used to photograph explosions through the 4-inch-diameter viewing port of a concrete bunker. Twenty-millimeter images of 150- to 450-mm explosive objects are required, with the objects 9000 mm from the front lens element. To obtain this range of magnification, a zoom ratio of approximately 3.6:1 is needed. Between the viewing port and the objective's image, there is a distance of more than 2000 mm, which can be used for any desired arrangement of lens elements.

In the search for a solution to this difficult imaging problem, the lens designs of US Patents 2,778,272, 3,051,052,

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PRESCRIPTION FROM DUMP NUMBER    1                    SAMPLE PROBLEM FOR ILLUSTRATION                    01--30--63

      0.0000000+00   -1.7951207-02   -6.6173003-02   -1.4243072-01   0.0000000+00   0.0000000+00   0.0000000+00
 10  -2.0000000+00    1.0000000-01    9.9999975-05    7.0699999-01   3.0000000+00   1.0183419+00   2.0000000+01
                                                           1.0000000+00  -1.0000000+00   9.9999975-05
 20   1.2000000+01    0.0000000+00    1.0000000+00    3.1000000+00   1.0000000+00   1.0000000+00   1.0000000+00
 30   0.0000000+00   -1.0133081+00    6.9734238+00    0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
 40   0.0000000+00    1.2500000+00   -1.5508849+00    0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
 50   0.0000000+00    5.0000000-01   -6.9003449+00    0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
 60   0.0000000+00    1.0000000+00    8.2007210+00    0.0000000+00   1.6510000+00   1.6477100+00   1.6588500+00
 70   0.0000000+00    1.2500000+00   -1.4668796+00    0.0000000+00   1.7200000+00   1.7143400+00   1.7342200+00
 80   0.0000000+00    5.0000000-01   -3.6679682+00    0.0000000+00   1.0000000+00   1.0000000+00   1.0000000+00
    PLANE SETTING MODE ONE.        LATTICE MODE ONE.         SENSE SWITCH MODE ZERO.        NUMBER OF PLANES   4

COLOR   FOCAL LENGTH        FOCAL POINT           H                  EXIT PUPIL        F/NUMBER        BACK FOCUS    RAYS

  1     3.4609916+00       1.9274388+00       0.0000000+00                            2.0000000+00    2.9934376+00    12

        LOC OF PLANE        AVERAGE X           AVERAGE Y              RMS X              RMS Y             SPOT SIZE
        2.8000000+00       0.0000000+00       -2.3685085-06       2.2382704-02       2.2390649-02       3.1652471-02
        2.9000000+00       0.0000000+00       -1.1044321-06       1.0950404-02       1.0849412-02       1.5344087-02
        3.0000000+00       0.0000000+00        1.5964423-07       1.3199177-03       1.3197630-03       1.8665503-03
        3.1000000+00       0.0000000+00        1.4237206-06       1.2370051-02       1.2368903-02       1.7493082-02

  2     3.4631695+00       1.9255137+00       0.0000000+00                            2.0034385+00    2.9896925+00    12

        2.8000000+00       0.0000000+00       -2.2860038-06       2.1875057-02       2.1873066-02       3.0934595-02
        2.9000000+00       0.0000000+00       -1.0217773-06       1.0356273-02       1.0355341-02       1.4645323-02
        3.0000000+00       0.0000000+00        2.4245432-07       1.7874692-03       1.7872633-03       2.5277176-03
        3.1000000+00       0.0000000+00        1.5066859-06       1.2897522-02       1.2896305-02       1.8238990-02

  3     3.4597524+00       1.9371255+00       0.0000000+00                            2.0024741+00    3.0116472+00    12

        2.8000000+00       0.0000000+00       -2.7098770-06       2.4660863-02       2.4658501-02       3.4874057-02
        2.9000000+00       0.0000000+00       -1.4494289-06       1.3125655-02       1.3124356-02       1.8561561-02
        3.0000000+00       0.0000000+00       -1.8898086-07       1.6848162-03       1.6845927-03       2.3825319-03
        3.1000000+00       0.0000000+00        1.0714672-06       9.9912712-03       9.9904499-03       1.4129211-02

  1                                            1.0000000+00      -3.6064031+00                                         12

        2.7820489+00       0.0000000+00        3.0654871-01       2.2792781-02       2.2360278-02       3.1929641-02
        2.8820489+00       0.0000000+00        3.1138678-01       1.1232371-02       1.0770165-02       1.5575425-02
        2.9820487+00       0.0000000+00        3.1622526-01       1.1078005-03       1.1907757-03       1.6263975-03
        3.0820487+00       0.0000000+00        3.2106354-01       1.2029262-02       1.2448006-02       1.7310576-02

  2                                            1.0000000+00      -3.6133179+00                                         12

        2.7820488+00       0.0000000+00        3.0680222-01       2.2275349-02       2.1820780-02       3.1182469-02
        2.8820488+00       0.0000000+00        3.1163812-01       1.0727079-02       1.0261464-02       1.4844793-02
        2.9820487+00       0.0000000+00        3.1647402-01       1.5204409-03       1.7160170-03       2.2926760-03
        3.0820487+00       0.0000000+00        3.2130992-01       1.2566453-02       1.3003705-02       1.8083475-02

  3                                            1.0000000+00      -3.5871504+00                                         12

        2.7820488+00       0.0000000+00        3.0602654-01       2.5094595-02       2.4719953-02       3.5225201-02
        2.8820488+00       0.0000000+00        3.1087417-01       1.3533551-02       1.3154644-02       1.8873306-02
        2.9820487+00       0.0000000+00        3.1572180-01       2.0354425-03       1.6369301-03       2.6120243-03
        3.0820487+00       0.0000000+00        3.2056943-01       9.6279156-03       9.9990360-03       1.3880831-02

  1                                            2.0000000+00      -3.6228702+00                                         12

        2.7338270+00       0.0000000+00        6.0558410-01       2.3494662-02       2.1891736-02       3.2113038-02
        2.8338270+00       0.0000000+00        6.1518953-01       1.1854614-02       1.0228542-02       1.5657577-02
        2.9338270+00       0.0000000+00        6.2479496-01       9.3775907-04       1.5360701-03       1.7996954-03
        3.0338269+00       0.0000000+00        6.3440039-01       1.1533753-02       1.3132303-02       1.7478124-02

  2                                            2.0000000+00      -3.6297793+00                                         12

        2.7338270+00       0.0000000+00        6.0609328-01       2.2947410-02       2.1255873-02       3.1279318-02
        2.8338270+00       0.0000000+00        6.1569399-01       1.1317571-02       9.5942936-03       1.4837045-02
        2.9338270+00       0.0000000+00        6.2529469-01       1.1650730-03       2.1590160-03       2.4533226-03
        3.0338269+00       0.0000000+00        6.3489540-01       1.2099325-02       1.3771157-02       1.8331351-02

  3                                            2.0000000+00      -3.6035930+00                                         12

        2.7338270+00       0.0000000+00        6.0453168-01       2.5867017-02       2.4500747-02       3.5628488-02
        2.8338270+00       0.0000000+00        6.1415581-01       1.4231919-02       1.2863325-02       1.9183656-02
        2.9338270+00       0.0000000+00        6.2377994-01       2.6287663-03       1.4107892-03       2.9834125-03
        3.0338269+00       0.0000000+00        6.3340407-01       9.0659417-03       1.0481516-02       1.3858336-02

  1                                            3.0000000+00      -3.6511035+00                                         12

        2.6575693+00       0.0000000+00        8.9051349-01       2.4571583-02       2.1429125-02       3.2603222-02
        2.7575693+00       0.0000000+00        9.0474763-01       1.2810747-02       9.7058506-03       1.6072450-02
        2.8575692+00       0.0000000+00        9.1898186-01       1.3191855-03       2.7113263-03       3.0152181-03
        2.9575692+00       0.0000000+00        9.3321609-01       1.0799184-02       1.4094348-02       1.7755929-02

  2                                            3.0000000+00      -3.6580822+00                                         12

        2.6575693+00       0.0000000+00        8.9128233-01       2.3975028-02       2.0627262-02       3.1627296-02
        2.7575693+00       0.0000000+00        9.0550951-01       1.2221248-02       8.8921410-03       1.5113471-02
        2.8575692+00       0.0000000+00        9.1973669-01       1.0790765-03       3.3028340-03       3.4745310-03
        2.9575692+00       0.0000000+00        9.3396387-01       1.1411046-02       1.4866034-02       1.8740673-02

  3                                            3.0000000+00      -3.6316620+00                                         12

        2.6575693+00       0.0000000+00        8.8371515-01       2.7062503-02       2.4464137-02       3.6481133-02
        2.7575693+00       0.0000000+00        9.0317744-01       1.5313304-02       1.2786176-02       1.9949526-02
        2.8575692+00       0.0000000+00        9.1743773-01       3.5945015-03       2.3193620-03       4.2778365-03
        2.9575692+00       0.0000000+00        9.3170202-01       8.2249543-03       1.1143313-02       1.3850029-02
```

Figure 4. Printed output showing Lister lens performance.

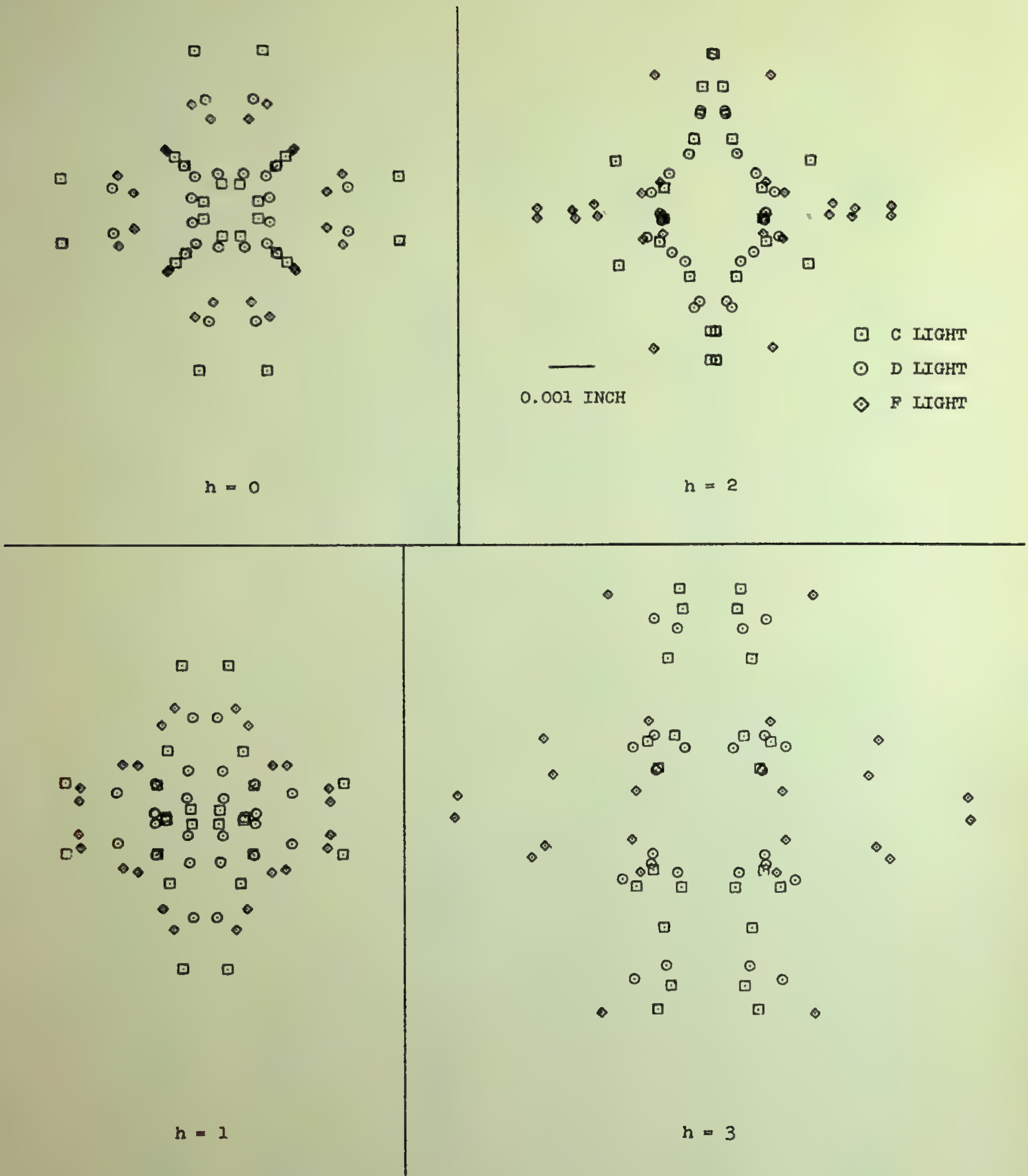

Figure 5. Spot diagrams of four images made by Lister lens.

and 3,057,259 were tested and all were found to be inadequate for our purposes because of insufficient correction of off-axis aberrations and/or excessive shift of the image over the zoom range. It was then decided to let the lens-designing program find the optimum design for lens systems with three or four or more elements so that we could select the simplest design which would give the required resolution. Using the program's substitution capability, we let the machine design each lens system simultaneously for five equally spaced positions of the moving elements. Several monochromatic designs were made with zoom arrangements which previous workers had found to be useful.[8] The so-called four-lens system, which has five elements, was found to be the minimum which would give the desired correction of all aberrations. The entire available length was used because a long lens nearly always gives the best performance. The design was then restarted with five cemented sheets of 517645 and 617366 glass (C, e, and g indexes) in a zoom system consisting of two coupled sliding lenses which move between three fixed lenses. These paired components quickly assumed shapes to give negative, positive, negative, positive, and positive powers. Before the final design was achieved, several readjustments of the movements and spacings of the elements were necessary.

The prescription data print is shown in Figure 6. Table 7

```
PRESCRIPTION FROM DUMP NUMBER  53                        BRIXNER ZOOM FOR 4/1 POSITION ONE.                          01--31--63

 10  -1.0000000+00  -9.9999999-03   1.0000000-05   7.0699999-01   3.7653234+02   5.4445989+01   1.0000000+00
                                                                  1.0000000+00  -2.0000000+01   9.9999999-07
 20   9.0000000+03   0.0000000+00   3.7500000+01   7.6000000+01   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
 30   0.0000000+00  -8.0000000+02   5.0464065+02   0.0000000+00   1.5152100+00   1.5177200+00   1.5196600+00
                                                                  1.5233500+00   1.5276900+00   1.5312600+00
 40   0.0000000+00   1.0000000+01   2.9585648+02   0.0000000+00   1.6117600+00   1.6167200+00   1.6206900+00
                                                                  1.6286000+00   1.6383900+00   1.6469200+00
 50   0.0000000+00   1.5000000+01   3.5059407+02   0.0000000+00   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
 60   0.0000000+00   1.0000000+01   3.5075321+02   0.0000000+00   1.5152100+00   1.5177200+00   1.5196600+00
                                                                  1.5233500+00   1.5276900+00   1.5312600+00
 70   0.0000000+00   2.0000000+01  -4.6271720+02   0.0000000+00   1.6117600+00   1.6167200+00   1.6206900+00
                                                                  1.6286000+00   1.6383900+00   1.6469200+00
 80   0.0000000+00   1.0000000+01   5.9126535+03   0.0000000+00   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
 90   0.0000000+00   5.0000000+02   4.5543400+04   0.0000000+00   1.5152100+00   1.5177200+00   1.5196600+00
                                                                  1.5233500+00   1.5276900+00   1.5312600+00
100   0.0000000+00   1.0000000+01   1.1846093+02   0.0000000+00   1.6117600+00   1.6167200+00   1.6206900+00
                                                                  1.6286000+00   1.6383900+00   1.6469200+00
110   0.0000000+00   1.5000000+01   2.1867210+02   0.0000000+00   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
120   0.0000000+00   1.0000000+01   5.8180050+02   0.0000000+00   1.5152100+00   1.5177200+00   1.5196600+00
                                                                  1.5233500+00   1.5276900+00   1.5312600+00
130   0.0000000+00   2.0000000+01  -2.2434079+02   0.0000000+00   1.6117600+00   1.6167200+00   1.6206900+00
                                                                  1.6286000+00   1.6383900+00   1.6469200+00
140   0.0000000+00   1.0000000+01  -6.7883659+02   0.0000000+00   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
150   0.0000000+00   1.0000000+03   4.4937513+02   0.0000000+00   1.5152100+00   1.5177200+00   1.5196600+00
                                                                  1.5233500+00   1.5276900+00   1.5312600+00
160   0.0000000+00   2.0000000+01  -2.2168650+02   0.0000000+00   1.6117600+00   1.6167200+00   1.6206900+00
                                                                  1.6286000+00   1.6383900+00   1.6469200+00
170   0.0000000+00   1.0000000+01  -6.4920066+02   0.0000000+00   1.0002060+00   1.0002060+00   1.0002060+00
                                                                  1.0002070+00   1.0002080+00   1.0002100+00
     PLANE SETTING MODE ONE.      LATTICE MODE ONE.       SENSE SWITCH MODE ZERO.       NUMBER OF PLANES   3

     WEIGHTS
     -49   1.2000000+01   8.0000000+02   0.0000000+00   1.0000000+02   0.0000000+00   0.0000000+00   0.0000000+00
     -39   1.0000000+02   1.0000000+02   1.0000000+02   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00
     -29   0.0000000+00   5.0000000+01   1.0000000+02   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00
     -19   0.0000000+00   0.0000000+00   1.0000000+02   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00
      -9   0.0000000+00   0.0000000+00   1.0000000+01   0.0000000+00   0.0000000+00   0.0000000+00   0.0000000+00

      LATTICE N =  6      DELTA Y =  5.0000000-01
               2.5000000-01  -7.5000000-01    4
               7.5000000-01  -2.5000000-01    2

     INCREMENT IDENTIFICATION...  1   18   12   10    8   9  10  11  12  13  14  17   1   4

     COLOR IDENTIFICATION...      3   3   5   1

                                     ADDRESSES OF DESIGN PARAMETERS         ACTUAL DET.    REQUIRED DET.    INCREMENT
      ID.    LOC.     NO.                                                   1.0001991+02   1.0000000+02    1.1339532-06
       1     1010     -1       32                                           1.0000682+02   1.0000000+02    5.7589209-06
       2     1020     -1       42                                           1.0465263+02   1.0000000+02    9.9999999-07
       3     1030     -1       52                                           1.0000476+02   1.0000000+02    1.0647986-06
       4     1040     -1       62                                           1.0000028+02   1.0000000+02    5.4958878-06
       5     1050     -1       72                                           1.1493105+02   1.0000000+02    9.9999999-07
       6     1060     -1       82                                           1.0000299+02   1.0000000+02    2.0959069-06
       7     1070     -1       92                                           1.0000603+02   1.0000000+02    1.0171679-05
       8     1090     -1      102                                           9.9994932+01   1.0000000+02    1.8061980-06
       9     1070     -1      112                                           1.0000235+02   1.0000000+02    1.5391297-06
      10     1100     -1      122                                           1.0000288+02   1.0000000+02    7.6109598-06
      11     1110     -1      132                                           9.9996854+01   1.0000000+02    1.2817393-06
      12     1120     -1      142                                           9.9997405+01   1.0000000+02    4.3133787-06
      13     1130     -1      152                                           1.0000758+02   1.0000000+02    2.2412374-05
      14     1140     -1      162                                           9.9989856+01   1.0000000+02    3.8367009-06
      15     1150     -1      172                                           1.0000774+00   1.0000000+00    6.4961967-02
      16     1160      1     1709                                           9.9991272-01   1.0000000+00    4.9294689-02
      17     1170      1     1809                                           9.9976323-01   1.0000000+00    8.5637964-02
      18     1180      1     1909                                           9.9976886-01   1.0000000+00    8.5473059-02
      19     1190      5       14   1602   1702   1802   1902

  SUBSTITUTIONS...     4 SUBSTITUTION SETS,  OF  11 ITEMS, EACH.
        LOC.     1ST SUBST.      2ND SUBST.      3RD SUBST.      4TH SUBST.
        -193   -8.8590784+06   -1.0835291+07    3.8407912+15    5.1604435+14
         -49    6.5000000+00    8.0000000+00    1.0000000+01    7.0000000+00
          14    3.7653234+02    3.7653234+02    3.7653234+02    3.7653234+02
          15    3.3457052+01    4.7129203+01    4.9695471+01    4.0865409+01
          20    9.0000000+03    9.0000000+03    9.0000000+03    9.0000000+03
          61    4.1000000+02    2.1000000+02    1.1000000+02    3.1000000+02
          91    1.0000000+02    3.0000000+02    4.0000000+02    2.0000000+02
         121    4.1000000+02    2.1000000+02    1.1000000+02    3.1000000+02
         151    6.0000000+02    8.0000000+02    9.0000000+02    7.0000000+02
          22    1.1250000+02    6.4928912+01    4.9282037+01    8.5562077+01
          23    2.2600000+02    1.7500000+02    1.2500000+02    2.0000000+02
```

Figure 6. Printed output showing final prescription of zoom lens.

shows selected optical characteristics of this lens for nine positions. For each of these nine positions the spot sizes and focal positions for C, d, e, F, g, and h light were determined. There was no discernible shift of the optimum focal plane for any of the positions tested, possibly because longitudinal chromatic aberration is the principal defect of this lens. Except for h light, the maximum spot sizes on the focal plane at the 384.709-mm back focus did not go above 0.027 mm, and the average is about 0.013 mm. A visual white-light resolution of about 100 lines/mm is predicted for most of the zoom range.

TABLE 7

OPTICAL CHARACTERISTICS OF ZOOM LENS

Zoom Position	Element Shift, mm	Focal Length, mm	Relative Aperture	Object Max. Size, mm
1	0	436	6.5	450
2	50	503	6.7	392
3	100	583	7.0	342
4	150	677	7.4	298
5	200	790	8.0	259
6	250	927	8.8	226
7	300	1096	10.0	197
8	350	1310	10.8	172
9	400	1590	12.0	150

CONCLUSION

This code has demonstrated its ability to analyze the performance of and/or to design simple or complex lens systems rapidly and precisely. Correlation between the lens performance predicted by the program's multiple ray-tracing analysis and the measured performance of the constructed lens has been close. When there has been an apparent discrepancy between measured and predicted performance, the cause has always been a manufacturing error. It is therefore necessary to give the manufacturer construction tolerances of sufficient tightness to insure that design specifications will be met. The construction-tolerance specifications can be determined by the program's ability to evaluate the effect of small parameter changes and the displacement and/or tilt of the surfaces. In this connection, particularly for complex designs, one needs precise values for the refractive indexes of the glasses (melts) used for making the lens elements, since the manufacturing tolerance for glass stock is greater than the tolerance allowable if predicted lens performance is to be obtained.

In conclusion, the code gives to problems of lens design a general automated solution which makes full use of the digital computer's capabilities and abandons the classical approach requiring direction at every stage by the skilled specialist.

REFERENCES

1. C. A. Lehman, LASL Lens Design Program for the IBM 7090 (Los Alamos Scientific Laboratory Report LA-2837, April, 1963).

2. John C. Holladay, "Computer Design of Optical-Lens Systems (IBM 704)" in Computer Applications-1960, Benjamin Mittman and Andrew Ungar, eds. (Macmillan Co., New York, 1961), pp. 112-127.

3. D. B. MacMillan and R. H. Stark, "'Floating Decimal' Calculation on the IBM Card-Programmed Electronic Calculator," IBM Tech. Newsletter (1951), No. 2, pp. 16-26.

4. William A. Allen and Richard H. Stark, "Ray Tracing Using the IBM Card-Programmed Electronic Calculator," J. Opt. Soc. Am. (September, 1951), Vol. 41, pp. 636-640.

5. Donald P. Feder, "Tracing of Skew Rays," J. Res. Nat. Bur. Std. (July, 1950), Vol. 45, pp. 61-63.

6. IBM General Information Manual 709-7090 Data Processing Systems (International Business Machines Corporation, Data Processing Division, White Plains, New York, August, 1960), Form D22-6508-2.

7. A. E. Conrady, Applied Optics and Optical Design, Rudolf Kingslake, ed. (Dover Publications, Inc., New York, 1960), Part Two, pp. 662-678.

8. Rudolf Kingslake, "The Development of the Zoom Lens," J. Soc. Motion Picture Television Engrs. (August, 1960), Vol. 69, pp. 534-544.